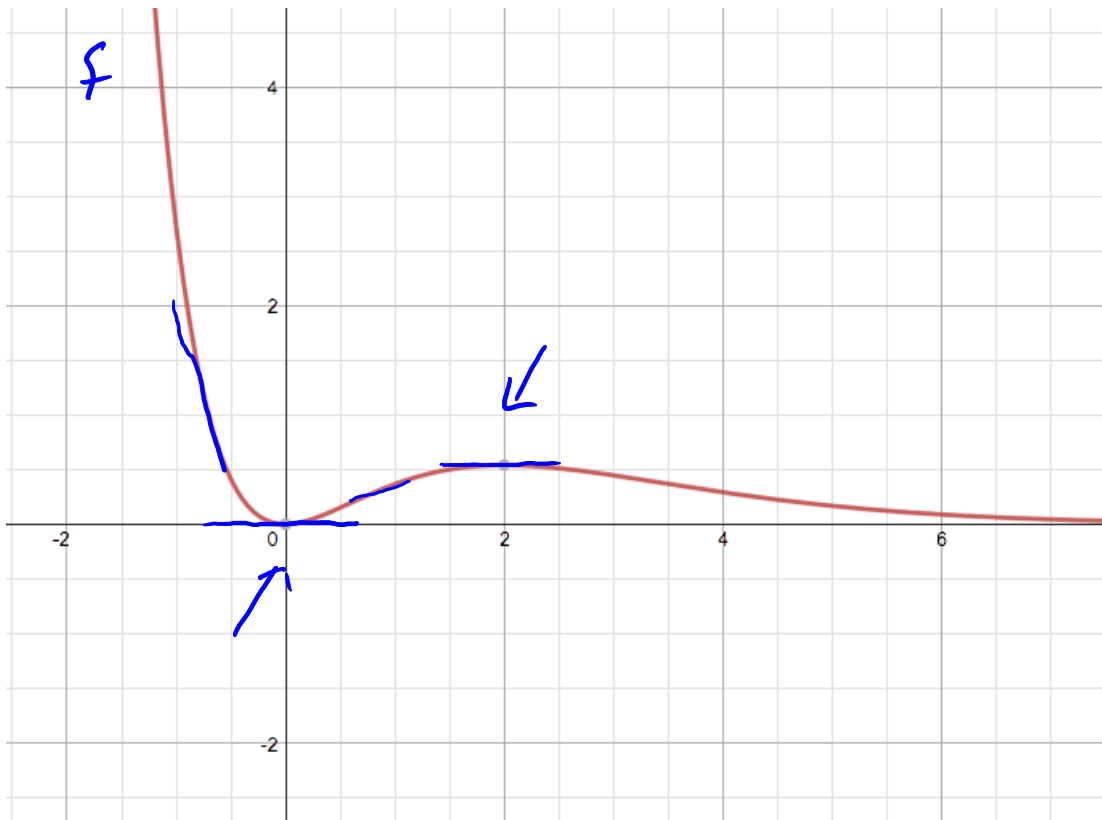


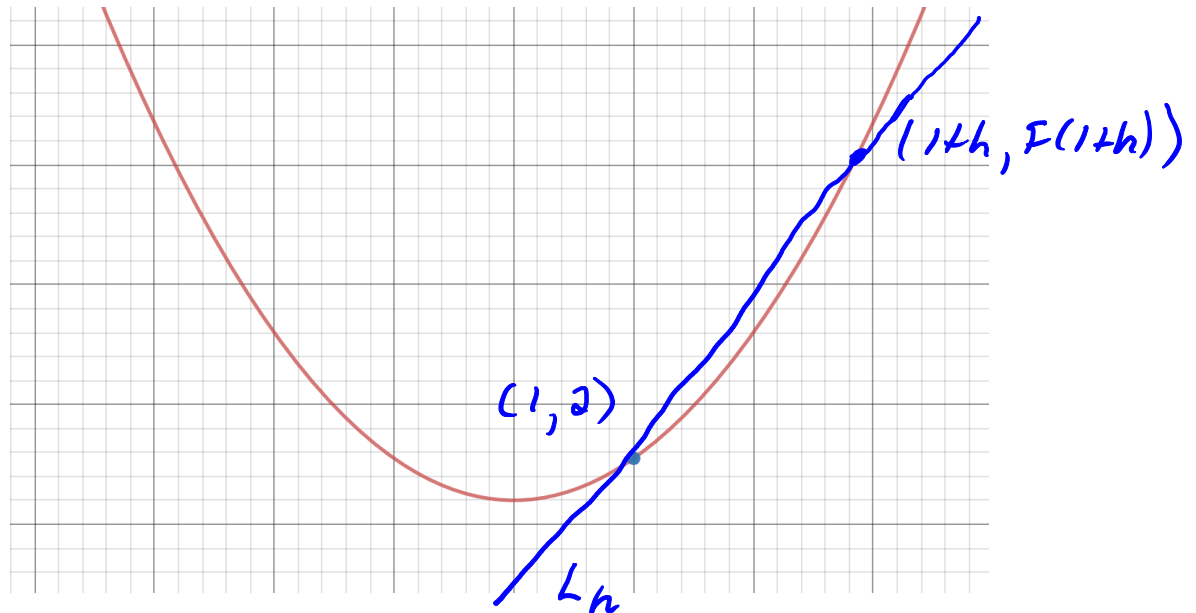
## Introduction to Calculus, Example 2 (Leibniz)

How can you determine the maximum and minimum values of a function? On what intervals is it increasing? On what intervals is it decreasing?



Suppose  $y = f(x) = x^2 + 1$ .

a) Sketch the graph of  $f$ .



b) Plot the points  $(1, f(1))$  and  $(1+h, f(1+h))$ .

c) Draw the line containing these two points.

d) Find the slope of this line (secant line)

$$(1, 2), (1+h, 1+2h+h^2+1)$$

$$(1, 2), (1+h, 2+2h+h^2)$$

$$\text{slope of } L_h = \frac{2+2h+h^2-2}{1+h-1}$$

$$= \frac{2h+h^2}{h} = 2+h \quad (\text{for } h \neq 0)$$

↑ As  $h \rightarrow 0$ ,

e) What happens to the slope when  $h$  is closer and closer to zero?

slope  $\rightarrow 2$

$2 =$  slope of the tangent line at  $(1,2)$

f) Find the equation of the line tangent to the graph of  $f$  at the point  $(1,2)$ .

$$y - 2 = 2(x - 1)$$

$$y = 2x - 2 + 2, \quad \boxed{y = 2x} \cdot$$

